Placed Rock as Protection against Erosion by Flow down Steep Slopes

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Abstract: The additional resistance to erosion due to flows down steep slopes that can be achieved by placing rock instead of dumping randomly has been quantified during a large-scale flume investigation. Testing was undertaken for slopes of 0.2, 0.3, and 0.4 with two layers of armor overlying a filter, consistent with conventional rock protection designs. Two sizes of sandstone with mean diameters of 76 and 109 mm and basalt with a mean diameter of 94 mm were tested. Placing rock to achieve maximum bulk density (mass per unit in situ volume) increased failure flow (flow at exposure of the filter) by 30% of that achieved with the same type of randomly dumped material but the total armor mass per unit surface area increased by 35%. A damage approach to steep erosion protection that is similar to that used for coastal breakwater design is presented. An improved method of partition between the aerated flow over as well as flow through both random and placed rock has also been quantified. Better characterization of aerated surface flow descriptions for random and placed crushed rock is presented. A non-Darcy coefficient of 3.3 ± 1.1 best characterized the interflow component through random and placed rock.

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Introduction

Rock protection against erosion by flow down steep slopes usually takes a constructed form of armor overlying crushed filter material with geotextile laid over the embankment prior to filter placement. Peirson and Cameron (2006) have shown that an appropriate modification to the stability equation of Hartung and Scheuerlein (1970) collapses available large-scale data, exhibits the correct dependency on slope, and provides a conservative design method.

No large-scale investigation has reported flow-depth relationships for randomly dumped rock that are suitable for checking the Hartung and Scheuerlein (1970) results which were obtained using fixed rock beds with a specific alignment of the armor material. In addition, no previous investigations have specifically addressed the potential benefits of rock placement with reduced porosity. Consequently, a large-scale laboratory investigation has been undertaken focussed on the following unresolved issues:

1. Codell et al. (1990) and Robinson et al. (1998) have shown that a proportion of the total discharge can be carried within the blanket of dumped rock armor (the so-called interflow). Is the interflow through placed rock significantly different from that characteristic of randomly dumped material?
2. Stephenson (1979, p. 51) includes reduced armor porosity as a stabilizing factor in his design method, but widely graded materials (with consequent lower porosities) are less stable (Abt and Johnson 1991) than those consisting of a narrow range of sizes. For material with low coefficients of uniformity [defined by Abt and Johnson (1991) as $d_{60}/d_{10}<2.3$, where d_x is size of graded rock with x% finer by mass], the effect on the failure discharge of reducing rock porosity by placement of the armor remains unknown.
3. Coastal breakwater design recognizes the costs of relocating armor moved during storms over the life of a structure. Could such an approach be applied to the flow erosion of rock armor on steep slopes?

Methodology

Armor Characterization

Two sizes of sandstone and a single size of crushed basalt were obtained from local quarries and sorted to ensure a coefficient of uniformity less than 2.3. After transport from quarry to test location, the sandstone appeared to be significantly less angular than the basalt. Each rock sample was carefully tested to obtain the characteristics summarized in Table 1.

Random dumping was achieved by dropping then scraping rocks along the slope with the objective of achieving a similar physical arrangement to larger scale material dumped using construction equipment. Placement was achieved manually by carefully arranging the rocks to minimize the porosity of the armor blanket. The average thickness of the two rock layers from the top of any underlying filter to the top of the armor was $1.60d_{50}$ to

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Table 1. Summary of Rock Properties

		Sandstone (d_{50}=76 mm)		Sandstone (d_{50}=109 mm)		Basalt (d_{50}=94 mm)	
Rock unit properties							
Specific gravity ($\pm 2\%$)	γ	2.29		2.37		2.64	
Mean diameter [mm]	d_{50}	76		109		94	
Coefficient of uniformity	C_u	1.18		1.14		1.21	
Rock arrangement properties							
Configuration		Random	Placed	Random	Placed	Random	Placed
Packing number [rocks/m^2/layer] ($\pm 5\%$)		145	187	71	90	93	117
Porosity (+2%)		0.47	0.39	0.44	0.38	0.47	0.37
Depth of two layers [mm] ($\pm 5\%$)		121	141	174	204	151	184
Unit density of two layer rock mass [kg/m^3] ($\pm 3\%$)		1,230	1,410	1,320	1,450	1,405	1,717
Friction angle [°]		44	49	50	49	48	53

$1.86d_{50}$ for the random and placed configurations, respectively. This was due to the reduced porosity of each layer of placed rock which reduced the size of overlapping voids, whereby two placed layers can interlock.

Rock specific gravity and an average armor porosity (n) were determined volumetrically. Dividing the mass of rock per square meter by the average depth of the two rock layers yielded a dry unit density and an alternative value of n. The two independently-determined values of n agreed within 2%. The angle of inner friction was determined from averaging three observations of the angle of tilt required to induce armor motion in a test ramp with an underlayer of 40 mm basalt fixed to the ramp floor.

Flow Testing

The flow test facility used was a tilting flume of 600 mm depth and 900 mm width. Depending on the test slope, the total flume length could vary between 4.2 m for a 0.40 slope up to 8.4 m for slopes more gentle than 0.20. The armor was installed in two layers on a filter consisting of two layers of 40 mm basalt with the lower filter layer fixed to the flume floor. Manometer tappings were located along the center line of the flume bed. Flow was measured by a 150 mm Yamatake–Honeywell electromagnetic flowmeter with a maximum error of 2%. The overall test sequence was as follows:

1. The flume was configured for the desired test slope and the test armor material was selected.
2. A second filter layer of 40 mm basalt was spread on a 40 mm basalt base fastened to the bed to give an average filter thickness of 70 mm.
3. Two layers of armor rocks were dumped or placed at the specified packing density.
4. A small flow ($<3\ l\text{s}^{-1}$) was applied and air in the manometer tubes was eliminated. The flow was turned off and all manometer tubes were zeroed for subsequent testing.
5. The discharge was increased until the filter layer was just submerged. It was found that this occurred at a flow of about $5\ l\text{s}^{-1}$.
6. Flow was increased in increments of $\sim 10\ l\text{s}^{-1}$. The total discharge and all manometer readings were recorded. Once the armor was submerged, the armor blanket was carefully monitored to detect movement of rocks within the armor blanket.
7. Testing was continued until failure of the rock armor exposed the filter material.

Results and Discussion

Flow Partitioning

The total discharge (q_{tot}) is the sum of two components: flow carried within the rock armor blanket itself (q_{int}, Codell et al. 1990) and flow over the armor (q_{over}). As the rock armor thickness increases, so also will the amount of flow that can be carried within the armor Codell et al. (1990) elected to partition the flow at the upper surface of the rock armor, but the upper layer of armor contains the roughness elements resisting the overflow and flow within this layer could be included in the overflow.

Peirson and Cameron (2006) showed that flow aeration is critical to an accurate characterization of the armor stability. Until an improved aerated flow characterization is available, the relationships determined by Scheuerlein (1968) should be retained and, for this study, the flow was partitioned consistent with Scheuerlein's work. Therefore, the base of the upper armor layer was used as the partition level between the interflow and the overflow, and flow occurring beneath the base of the upper layer of armor was, thus, defined to be interflow during this study.

Interflow Characterization

Non-Darcy flow can be expressed (e.g., Stephenson 1979, p. 50) in the form

$$i = K_t V_{\text{int}}^2/(d_{50} g n^2) \tag{1}$$

where i is the hydraulic gradient (found to be identical to the embankment slope except within 0.6 m of the ends of the test rig); K_t is the non-Darcy flow parameter, and V_{int} is the bulk interflow velocity. Li et al. (1998) review alternative formulations and potential sources of variation in K_t and the interflow velocity exponent.

Manometer readings were taken as the flow was incrementally increased from the point of submergence of the filter to the point where the depth reached $d_{50}/2$ below the top of the armor. After deducting the flow observed through the filter from the total, a stable bulk velocity in the armor was obtained across a range of interflow depths. For all test conditions, the occurrence of turbulent (non-Darcy) flow within the armor was confirmed by high pore Reynolds numbers $[\mathsf{R}_p \approx V_{\text{int}} n d_{50}/6(1-n)\nu > 200]$, where ν is the kinematic viscosity of water. The derived non-Darcy coef-

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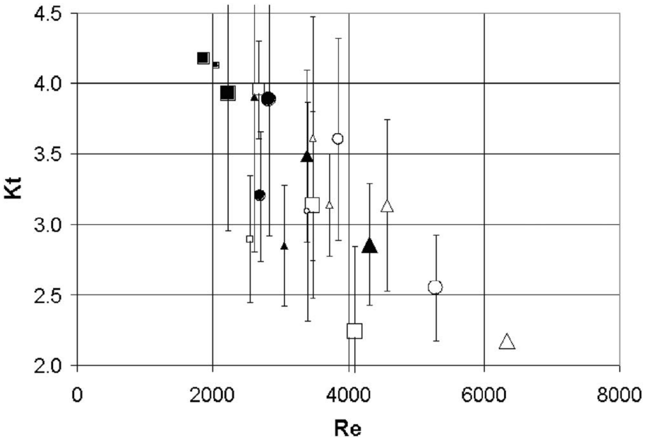

Fig. 1. Non-Darcy flow coefficients as a function of pore Reynolds number. Symbols indicate the test materials: squares, 76 mm sandstone; triangles, 109 mm sandstone; circles, 94 mm basalt. Hollow symbols indicate random armor, filled symbols indicate placed armor. Symbol size indicates the test slope $\tan\theta$: smallest symbols, 0.2; medium-sized symbols, 0.3; largest symbols, 0.4.

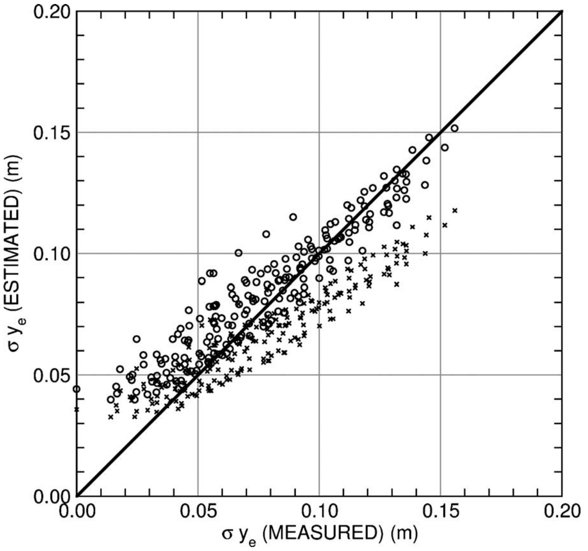

Fig. 2. Comparison between measured depths and those computed using Eqs. (2) and (6). The crosses and circles indicate the results for values of $A=3.2$ and 1.8, respectively. There were no observable systematic differences in overflow behavior between the random and placed armor.

ficients [Eq. (1)] are shown in Fig. 1, where the error bars indicate the standard deviation of the coefficient estimates obtained from the mean armor bulk velocity measurements.

The following observations can be made regarding the results shown in Fig. 1:

1. The K_t values obtained are tightly clustered in comparison with previous measurements (cf. Stephenson 1979, Fig. 2.2).
2. No systematic differences in K_t can be observed between the sandstone and the basalt although the sandstone appeared to be significantly less angular.
3. The mean non-Darcy coefficient obtained was 3.3 ± 1.1 for the entire data set with an increase in mean coefficient from 3.1 to 3.5 from random to placed material.
4. The test data span the practical range of achievable in situ porosities. The non-Darcy coefficients associated with placed rock are systematically higher than the corresponding values obtained for the randomly dumped material. This indicates that the porosity exponent of 2 in Eq. (1) is not sufficiently high if the non-Darcy flow model of Eq. (1) is assumed.

The differences observed in K_t between placed and randomly dumped material raise questions about previous measurements of non-Darcy coefficients, which are generally obtained by permeameter head loss measurements (e.g., Dudgeon 1966). Further investigation of the potential role of rock placement in the determination of K_t is warranted.

Overflow

There are insufficient measurements of flow depth available in the open literature to independently verify the coupled equations of Scheuerlien (1968) for rock material that is not fixed to the bed. The equations, with some minor simplification, are as follows:

A continuity equation incorporating aeration

$$q_{\text{over}} = \sigma y_e v_e \tag{2}$$

where q_{over} is the total overflow discharge per unit width, σ is the mean volumetric proportion of water in the air-water mixture, v_e is a representative mean velocity of the air-water mixture, and the representative flow depth y_e

$$y_e = y_0 + d_{50}/3 \tag{3}$$

y_0 is the depth above the armor, and $d_{50}/3$ is the characteristic hydraulic roughness.

Equilibrium between fluid and bed drag

$$v_e = \sqrt{8g/f}\sqrt{y_e \sin\theta} \tag{4}$$

where θ is channel slope and f is a Darcy friction coefficient.

Empirical equations for σ and f applicable to fixed rock beds

$$\sigma = 1 - 1.3\sin\theta + 0.24 y_e/d_{50} \tag{5}$$

$$\frac{1}{\sqrt{f}} = -A\log_{10}[\sigma d_{50}(1.7 + 8.1\Phi_2\sin\theta)/(12y_e)] \tag{6}$$

where $\Phi_2=1$ is the ratio of rock height above the bed to the mean distance between the centers of mass of adjacent rocks.

Hartung and Scheuerlein (1970) recommended $A=3.2$ in spite of Scheuerlein (1968) quoting values of A ranging from 1.8 to 2.5. In Fig. 2, the predicted flow depths obtained from Eqs. (2)–(6) are compared with the measured values. $A=3.2$ systematically underpredicts flow depth close to failure and for crushed material $A=1.8$ is able to predict flow depth to within $\pm 20\%$ if y_0 is greater than d_{50}.

Randomly Dumped Armor Failure

Rock armor failure, defined as exposure of the filter below the armor, is the conventional basis for protection design (e.g., Abt and Johnson 1991). With A changed from 3.2 to 1.8 in Eq. (6), the stability equation coefficient of Peirson and Cameron (2006) for failure of random dumped rock must be revised to

$$v_e = 0.88\sqrt{2g(\rho_s-\rho)/\sigma\rho}\sqrt{d_{50}\cos\theta}\sqrt{\tan\varphi - \tan\theta} \tag{7}$$

where φ=angle of friction of the rock; ρ=density of water; and ρ_s=density of the rock. The revised coupled system of equations [Eqs. (2)–(7)] has been applied to the three types of randomly dumped rock investigated during the present study (shown in

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Table 2. Summary of Study Results

	Sandstone (d_{50}=76 mm)			Sandstone (d_{50}=109 mm)			Basalt (d_{50}=94 mm)		
tan θ	0.20	0.30	0.40	0.20	0.30	0.40	0.20	0.30	0.40
Random									
1. $q_{\mathrm{tot,fail}}$ [$\mathrm{m}^2\ \mathrm{s}^{-1}$]	0.118	0.084	0.054 ± 0.03	0.233	0.156	0.156	0.194	0.183	0.161
2. $q_{\mathrm{over,fail}}/q_{\mathrm{over,fail}}$ (predicted)	1.42	1.32	1.05 ± 0.05	1.21	1.02	1.30	1.14	1.40	1.58
3. $q_{\mathrm{init}}/q_{\mathrm{over,fail}}$	0.35	0.50	0.47 ± 0.26	0.29	0.47	0.62	0.35	0.46	0.53
4. $q_{\mathrm{sig}}/q_{\mathrm{over,fail}}$	0.50	0.64	0.80 ± 0.13	0.51	0.77	0.85	0.50	0.56	0.78
Placed									
5. $q_{\mathrm{over,fail}}$ (placed)/$q_{\mathrm{over,fail}}$ (random)	1.32	1.75	2.29	1.32 ± 0.04	1.57	1.29	Not avail.	1.48	1.59
6. $q_{\mathrm{init}}/q_{\mathrm{over,fail}}$	0.37	0.39	0.47	0.51 ± 0.07	0.60	0.24	Not avail.	0.42	0.42
7. $q_{\mathrm{sig}}/q_{\mathrm{over,fail}}$	0.52	0.57	0.90	0.55 ± 0.07	0.79	0.94	Not avail.	0.73	0.82

Note: Where repeat measurements were undertaken, the mean value and the range are shown.

Table 2, rows 1 and 2) as well as the more extensive data set assembled in Peirson and Cameron (2006). The estimated overflow discharges at failure are conservative. A repeatability test was undertaken for random armor using the smaller sandstone material at tan θ=0.4. For this case, the mean of two experiments is reported and the variation from the mean indicated.

Assuming an angle of friction of rock of 40° and nondimensionalizing the estimated overflow failure discharge as $q_{\mathrm{over,fail}}\sqrt{\rho/(g(\rho_s-\rho)d_{50}^3)}$ collapses Eqs. (2)–(7) to the form

$$q_{\mathrm{over,fail}}\sqrt{\rho/(g(\rho_s-\rho)d_{50}^3)} = B10^{-1.37\tan\theta} \tag{8}$$

although changing rock density from 2.3 to 2.7 $\mathrm{kg.m}^{-3}$ modifies the coefficient B from 1.60 to 1.76. The depth of flow above the rock armor ($y_{0,\mathrm{fail}}$) at failure can be represented by

$$y_{0,\mathrm{fail}}/d_{50} = C10^{-0.895\tan\theta} \tag{9}$$

the coefficient C being 0.85 or 0.98 for rock densities of 2.3 and 2.7 $\mathrm{kg\ m}^{-3}$, respectively.

Aeration is primarily a function of Froude number and independent of Reynolds number above a threshold Reynolds number of approximately 70,000 [see discussion in Peirson and Cameron 2006]. Provided geometric and material density similarity between different scales exists, the experiments can be extrapolated to larger scale conditions via Froude scaling.

Placed Armor Failure

The parameterization of Stephenson (1979, Eq. 3.34, p. 51) predicts that rock armor stability is increased if armor porosity is reduced. In terms of the design/construction processes of large armored slopes, it is difficult for designers to specify porosity as a construction requirement, as it is a relatively difficult concept to communicate to equipment operators and difficult to determine under field conditions. During this present investigation, an alternative approach has been used which is better suited to the needs of the design/construction process. Those constructing the laboratory slopes were instructed to pack the armor so that spaces between armor were minimized. The mean porosities and armor thicknesses achieved in the packed configuration are shown in Table 1.

Table 2, row 5 shows the failure overflow for placed armor as a ratio of the corresponding failure overflow of randomly dumped rock. Increases in failure flow greater than a factor of 2 were observed specifically for single case of the smaller sandstone material at a slope of 0.4. Otherwise, the overall results indicate only modest benefit in the use of placed material. On the basis of the present results, a reliable increase of only 30% in the design flow can be obtained with a shift from random to placed material. However, this is only achieved with a corresponding increase in the volume of rock material of at least 35%. Allowing for the significantly higher placement costs, it is only in cases where larger material is very expensive to obtain that the use of placed armor could be economically justified.

Early in the test program, a repeatability test was undertaken for placed armor using the larger sandstone material at tan θ =0.2. The mean of two experiments is reported and the variation from the mean indicated. When packed, the smallest material tested did achieve significantly higher failure overflow values at the higher slopes. Further investigation of the potential for like configurations to yield economic designs may be warranted.

The design equation provided by Stephenson (1979, Eq. 2.2) suggests an increase in stability that is directly proportional to the decrease in porosity but these present results show that greater stability by packing can be achieved. Placement for minimum porosity couples the rock blanket together mechanically in a way that is not significantly achieved by random dumping. Consequently, the observed increase in the flow required to induce instability can be attributed to mechanical interlocking—a process not previously incorporated in rock armor design methods.

Development of Armor Instability

The concept of levels of armor instability are well established in engineering design procedures for rock protection against coastal storm wave attack (Jackson 1968). Abt and Johnson (1991) considered the effects of initial stone movement and channelization. Economic management of large armored slopes requires more detailed quantification of damage levels.

The following definitions of damage were adopted with the associated overflows causing such instability as indicated in the brackets:

1. Initial displacement of a single stone anywhere on the test surface (q_{init}).
2. Significant rock motion, defined as displacement of five rocks over a distance of more than 5 diameters (q_{sig}).
3. Armor failure, that is, exposure of the filter layer ($q_{\mathrm{over,fail}}$).

Table 2, rows 3, 4, 6, and 7 show the quantities q_{init} and q_{sig} as a proportion of $q_{\mathrm{over,fail}}$. Values of $q_{\mathrm{init}}/q_{\mathrm{over,fail}}$ and $q_{\mathrm{sig}}/q_{\mathrm{over,fail}}$ of

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30 and 50%, respectively are appropriate for design. These results do quantify the decreasing ductility of the armor instability processes with increasing armor slope. At a steep slope of 0.40, only a small increment in flow ($\sim 6-22\%$) was required for the filter to be exposed, after significant motion of the armor had occurred. In contrast, at a lower slope of 0.20, the required flow increment to move from significant rock displacement to failure was close to 100%. It was observed that at lower slopes, armor units would be initially displaced to form a more stable armored bed of variable slope similar to a pool-riffle system.

The stone movement ratios obtained by Abt and Johnson (1991) at lower slopes are similar to the present $q_{\mathrm{sig}}/q_{\mathrm{over,fail}}$ values rather than $q_{\mathrm{init}}/q_{\mathrm{over,fail}}$. The channelization process described by Abt and Johnson (1991) for more gentle slopes was not investigated specifically during this study as it was not observed during the rapid failure of these steep slopes.

Conclusions and Recommendations

The potential for interflow and overflow contributions to flow through and down two layers of rock armor on steep embankments has been investigated in terms of a partition using the non-Darcy flow equation of Stephenson (1979) and aerated flow characterization of Scheuerlein (1968). The set of equations for design have the flow partitioned at the base of the upper layer of two layers of rock armor.

Interflow measurements showed that the mean non-Darcy coefficient increased from 3.1 to 3.5 for random and place configurations, respectively. There was little evidence of systematic difference in non-Darcy coefficient between angular and semi-rounded rock. The porosity term in the Stephenson (1979) non-Darcy flow [Eq. (1)] did not adequately capture the effect of changing porosity between randomly dumped and placed armor. Further investigation of the impacts of rock packing on the non-Darcy coefficient is recommended.

The overflow measurements show that reducing the value of the coefficient A (defined in the text) from 3.2 to 1.8 in the aerated flow equations of Scheuerlein (1968) better characterizes the flow depths observed above both randomly dumped and placed rock armor close to failure. For depths greater than d_{50}, predictions match the measurements with an accuracy better than $\pm 20\%$. As a result of this improved hydraulic characterization, a slight modification of the coefficient in the rock armor stability equation [Eq. (7)] for randomly dumped rock has been recommended.

The additional resistance to erosion due to flows down steep slopes that can be achieved by placing rock instead of randomly dumping has been quantified. It has been found that placing rock to minimize the armor porosity reliably increases the failure flow beyond that of randomly dumped material by approximately 30%, but the mass per unit area increases by 35%. Two test conditions using the smallest test material did yield significantly higher increases in failure flow when packed.

During this present investigation, a damage approach to steep erosion protection is presented that is similar to that used for coastal breakwater design. For both random and placed armor, the flow must not exceed 30% of the failure flow for any movement of armor to be avoided. Flow must not exceed 50% of the failure flow to avoid substantial rearrangement of the armor on the slope—although this factor does systematically increase with embankment slope.

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Notation

The following symbols are used in this technical note:

A = coefficient in friction equation;
B = coefficient in a normalized equation for design failure overflow;
C = coefficient in a normalized equation for design failure depth;
C_u = coefficient of uniformity ($=d_{60}/d_{10}$);
d_x = size of graded rock with x% finer by mass;
f = Darcy friction coefficient;
g = gravitational acceleration;
i = hydraulic gradient;
k_s = mean roughness height of the stones;
K_t = non-Darcy flow parameter;
n = the porosity;
q_{tot} = total measured volumetric flow per unit width without entrained air;
q_{init} = overflow to initiate displacement of a single stone anywhere on the test surface;
q_{int} = interflow;
q_{over} = volumetric overflow to per unit width without entrained air;
$q_{\mathrm{over,fail}}$ = overflow to initiate armor failure (exposure of the filter layer);
q_{sig} = overflow to initiate significant rock motion (displacement of five rocks over a distance of more than 5 diameter);
R = Reynolds number ($=q/\nu$);
R_p = pore Reynolds number;
ν_e = representative mean velocity of the air-water mixture;
V_{int} = bulk interflow velocity;
y_e = representative flow depth above the base of the upper layer of armor;
y_0 = representative flow depth above the upper layer of armor;
$y_{0,\mathrm{fail}}$ = representative flow depth above the upper layer of armor at failure;
θ = angle of the slope;
ν = kinematic viscosity (assumed to be $1\times10^{-6}\ \mathrm{m^2\ s^{-1}}$);
ρ = density of water without entrained air;
ρ_s = rock density;
σ = the mean volume of water per unit mixture volume;
τ_0 = bed-shear stress;
φ = angle of friction of the rock material; and
Φ_2 = rock packing factor.

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