JOURNAL OF THE AMERICAN WATER RESOURCES ASSOCIATION

AMERICAN WATER RESOURCES ASSOCIATION

DISCUSSION[1]

"Evaluation of Overtopping Riprap Design Relationships" by Steven R. Abt, Christopher I. Thornton, Bryan A. Scholl, and Theodore R. Bender[2]

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Pells, Steven E. and William L. Peirson, 2014. Discussion — "Evaluation of Overtopping Riprap Design Relationships" by Steven R. Abt, Christopher I. Thornton, Bryan A. Scholl, and Theodore R. Bender. *Journal of the American Water Resources Association* (JAWRA) 1-4. DOI: 10.1111/jawr.12260

The authors (Abt *et al.*, 2013) have brought together a broad representation of published rip-rap stability equations. Each formulation was assigned merit based on the ability to reproduce selected laboratory test results. No distinction is made between empirical and analytical methods, and Abt *et al.* (2013) do not assess the merit of equations in representing physical processes. Empirical equations can only be assumed to be effective when used in conditions nearly identical to the conditions used to provide data to create the equation. Formulations that correctly recognize important physical processes must provide better predictions. For example, aeration processes which occur at large scales may not be represented in an empirical fit to laboratory data, yet are known to change hydraulic behavior considerably.

This discussion paper provides a summary of the logic underpinning those analytical formulations presented in Abt *et al.* (2013). The 21 equations presented by Abt *et al.* (2013) can be distilled into: 9 equations which are based on empiricism/dimensional analysis (Abt and Johnson, 1991; Maynord, 1994; Frizell *et al.*, 1998; Robinson *et al.*, 1998; Ferro, 2000; Natural Resources Conservation Service, 2002; Siebel, 2007; Eli and Gray, 2008; Khan and Ahmad, 2011) and 11 that have an analytical basis (Isbash, 1936; Olivier, 1967; Hartung and Scheuerlein, 1970; Knauss, 1979; Stephenson, 1979; Brown and Clyde, 1989; Chang, 1998; Mishra, 1998; Lagasse *et al.*, 2006; Peirson and Cameron, 2006; Peirson *et al.*, 2008) (the remaining publication, Wittler and Abt, 1997 was unable to be obtained by the writers). Of the analytical equations, all are based on consideration of freebody forces on a particle (e.g., Figure 1) in sliding, which is assessed by balancing the acting forces as per Equation (1) with resisting forces as per Equation (2), such that the critical drag force required to initiate movement, in all cases, is given by Equation (3) (note: θ is the slope angle, and ϕ is the angle of internal friction of the rockfill, such that $\tan\phi$ is a "coefficient of friction").

$$\sum F_A = W_{\text{sub}}\sin\theta + F_D \tag{1}$$

$$\sum F_R = W_{\text{sub}}\cos\theta\ \tan\phi \tag{2}$$

$$\sum F_{D,\text{crit}} = W_{\text{sub}}\cos\theta(\tan\phi - \tan\theta) \tag{3}$$

This is the fundamental stability condition assumed by these authors. It is also assumed in sediment transport, albeit with an inclusion of lift force (e.g., Ikeda, 1982; Wiberg and Smith, 1987; Chiew and Parker, 1994). The differences observed in the

[1]Discussion No. JAWRA-14-0010-D of the *Journal of the American Water Resources Association* (JAWRA). DOI: 10.1111/jawr.12260

[2]Paper No. JAWRA-12-0095-P of the *Journal of the American Water Resources Association* (JAWRA) 49(4):923-937.

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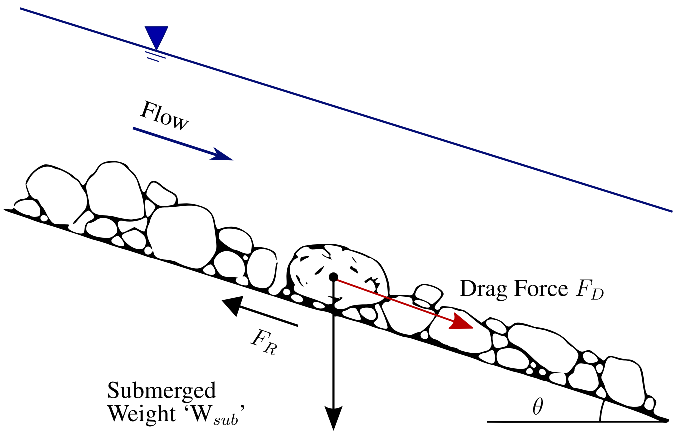

FIGURE 1. Typical Rip-Rap Stability Freebody Diagram as Presented in Various Hydraulics Publications.

published equations by these authors as presented in table 2 of Abt *et al.* (2013) reflect the choice of hydraulic roughness equations made by each author, as required to backcalculate the flow conditions associated with the critical drag condition. If we allow practitioners to utilize their preferred roughness equation, the Equation (3) above can be used in lieu of the diverse published methods.

However, while formerly extolling the virtues of analytical equations, three sobering observations can be made about this particular widely adopted equation.

Firstly, these analytical solutions hinge on a linear empirical constant. The average drag force, in Equation (3), is estimated (or inferred by the choice of a Shield's entrainment number, as shown below) in the cited publications in one of two ways: the classical "drag equation" ($F_D \cong C_D \rho \frac{u^2}{2} \cdot A$) or the "tractive force" equation ($F_D \cong \bar{\tau}_0 \cdot A$). In either case, the assessed critical drag force is calibrated against an assumed value of "A". Where the drag force is calculated as a proportion of tractive force, and the area "A" is taken as $A = c_1 \cdot D^2$ (D is the diameter and, e.g., if $c_1 = \frac{\pi}{4}$, "A" is the surface area of one quarter of a sphere) and the volume of the element $= c_2 \cdot D^3$ (e.g., $c_2 = \frac{\pi}{6}$, gives the volume of a sphere) then Equation (3) becomes:

$$\psi_{\text{crit}} = \frac{c_2}{c_1} \cos\theta(\tan\phi - \tan\theta) \quad (\psi_{\text{crit}} = \text{the Shield's entrainment number}) \tag{4}$$

The analytical equations of the form of Equation (3) therefore reduce to a Shields stability criterion, calculated using a linear empirical constant to factor the tractive force to the particulate level. With respect to the nonlinear shape of the Shield's diagram, a single linear factor is not enough to be representative of various hydraulic conditions (this is discussed in Henderson, 1966; Chapter 10.3) and this explains why such a disparate range of c_1 factors are adopted by various authors (see Table 1), and perhaps why, like empirical solutions, many published analytical equations were found by Abt *et al.* (2013) to be trustworthy only at reproducing conditions against which they were calibrated.

Secondly, the freebody on which Equation (3) is based (e.g., Figure 1) is only representative of a

TABLE 1. Alternative Solutions to Equation (3) Presented in Literature.

Author	θ	F_D **Calculated as:**	**Shape Factors**
Isbash (1936)	—	$F_D = C_D \rho \frac{u^2}{2} c_1 d D^2$ #1	$\sqrt{\frac{c_2 \cos\theta(\tan\phi - \tan\theta)}{c_1 C_D}}$ =1.2 (sheltered stones) =0.86 (protruding stones)
Olivier (1967)	✓	$F_D = \bar{\tau}_0 \cdot c_1 D^2$	$c_1 = \frac{\pi}{4}$ to 1; $c_2 = \frac{\pi}{6}$
Hartung and Scheuerlein (1970)	✓	$F_D = \sigma \rho \frac{u^2}{2} c_1 d D^2$	$c_1 = \frac{\pi}{4}$; $c_2 = \frac{\pi}{6}$; σ is an aeration factor
Knauss (1979)	—	$F_D = \bar{\tau}_0 \cdot c_1 D^2$	$\frac{c_2}{c_1} \tan\phi = 0.07$ to 0.1
Stephenson (1979)	✓	$F_D = C_D \rho \frac{u^2}{2g} c_1 d D^2$ #2	$\frac{c_D c_1}{c_2} = 0.5$
Brown and Clyde (1989)	—	$F_D = \bar{\tau}_0 \cdot c_1 D^2$ #1	$\frac{c_2}{c_1} \tan\phi = 0.047 \cdot K_1$ #3
Chang (1998)	✓	$F_D = \bar{\tau}_0 \cdot c_1 D^2$	$\frac{c_2}{c_1} = \frac{1,810}{g(\rho_s - \rho)\tan\phi} D^{0.177}$
Mishra (1998); Lagasse *et al.* (2006)	✓	$F_D = \bar{\tau}_0 \cdot c_1 D^2$	$\frac{c_2}{c_1} = 0.047$ #4
Peirson and Cameron (2006)	✓	$F_D = \sigma \rho \frac{u^2}{2} c_1 d D^2$	$c_1 \approx \frac{\pi}{8}$; $c_2 = \frac{\pi}{6}$
Peirson *et al.* (2008)	✓	$F_D = \sigma \rho \frac{u^2}{2} c_1 d D^2$	$c_1 \approx \frac{3\pi}{7}$; $c_2 = \frac{\pi}{6}$

#1 Drag equation inferred.
#2 The "g" in denominator in Stephenson (1979) is considered to be erroneous.
#3 K_1 is a factor for side slopes = $\left[1 - \frac{\sin^2\beta}{\sin^2\phi}\right]^{0.5}$ (K_1 = 1 for flat beds).
#4 Lagasse *et al.* (2006) cites Mishra (1998), who in turn cites Whittaker and Jäeggi (1986) for their stability equation, which characterizes $\psi = \frac{\tau}{gD(\cos\theta\rho_s - \rho)\tan\phi} = 0.047$.

global failure mechanism (e.g., Peirson *et al.*, 2008). While the freebody shown in Figure 1 is drawn at the particulate level, it represents stability in sliding, and the rock analyzed can only move in sliding if its neighboring rock slides as well, and so forth throughout the slope. Furthermore, body forces from adjacent rocks can only be neglected, as they are in Figure 1, if elements on either side of the selected block are identical to those on the block up and down slope from it, and so forth continuing throughout the slope (this is a problem tackled in many soil mechanics texts as an "infinite slope" problem, e.g., Das, 1979). The empirical constant "A" attempts to factor hydraulic forces to the particulate level. Hence, Figure 1 mixes particulate hydraulic forces with a global rock mechanics stability criterion and, as such, violates a global force balance.

Thirdly, analytical equations of the form of Equation (3) include a conceptual error with respect to an assumption of buoyant weight. The geotechnical disciplines, in following the principle of effective stress, tend to discretize the forces of self-weight from hydraulic ("pore pressure") forces rather than assuming the lumped "buoyant" or "submerged" weight. This is done because, as the examples of "infinite slope" analysis presented in geotechnical texts demonstrate (e.g., Das, 1979), there are many conditions, depending on the direction of seepage flow within the body, under which the uplift due to pore pressures on an element is not equal to the mass of water displaced by the element. Happily, if seepage flow is assumed to be parallel to the slope, the resisting forces assessed in this manner agree with Equation (2). However, the "infinite slope" assessment adopts the full weight in characterizing acting forces. Equation (1) adopts the submerged weight (e.g., $W_{\mathrm{sub}}\sin\theta$) in characterizing acting forces, which is considered by the writer to be wrong. Reconciliation with the geotechnical "infinite slope" formulation is only found with inclusion of a correction "α":

$$\sum F_{D,\mathrm{crit}} = W_{\mathrm{sub}}\cos\theta(\tan\phi - \alpha\tan\theta), \quad \text{where } \alpha = \frac{\rho_s}{\rho_s - \rho} \tag{5}$$

The error in the typical "hydraulic" conceptualization therefore increases with slope. For an assumed slope with $\theta = 25$, $\phi = 30$, $\rho_\sigma = 2{,}650\ \mathrm{kg/m^3}$, and $D = 1$, the typical "hydraulic" conceptualization (Figure 1) indicates that a hydraulic shear force of 0.9 kN is required to initiate failure. The geotechnical Equation (5), however, correctly indicates that the slope has already failed (a restraining force of 1.3 kN is required for stability!). The presence of water, alone (i.e., without any drag force induced), reduces the effective stress and therefore also the earth shear stress such that the slope fails globally.

The broad array of rip-rap stability equations collated by Abt *et al.* (2013) may leave the reader with the impression that the world is saturated with alternatives for design equations. However, it is argued that there is yet need for formulations which adequately represent physical processes. A global stability equation, for equilibrium, must apply the full unfactored tractive force. A local (particulate level) solution, such as required to simulate scour holes, must represent complex inter-body forces as well as hydraulic uplift and turbulent fluctuations.

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