Notes on falling / rising head test analysis

1.1 Inflows

A shape factor 'F' can be used to described geometry into which groundwater flows, such that discharge'Q' is given by:

$$Q = FkH \tag{1.1.1}$$

Where k is the hydraulic conductivity of the formation and H is the head differential.

A shape factor for wells, such as the well shown in Figure 1.1 is presented in Hvorslev (1951) as:

$$F = \frac{2\pi L}{\ln\left(\frac{\mathrm{L}}{\mathrm{D}} + \sqrt{1 + \left(\frac{\mathrm{L}}{\mathrm{D}}\right)^2}\right)} \tag{1.1.2}$$

$$\approx \frac{2\pi L}{\ln\left(2\frac{\mathrm{L}}{\mathrm{D}}\right)}$$

$$\approx \frac{2\pi L}{\ln\left(\frac{\mathrm{L}}{\mathrm{R}}\right)} \tag{1.1.3}$$

Where R is the outside radius of the drill hole (ie outside of any gravel pack etc).

As such, the discharge into the well in Figure 1.1 can be estimated, and it can be seen that discharge is linearly related to the head 'H' (the water level in the well remains above the screen, so the screen length and shape factor are constant).

It should be noted that there is no drawdown curve in Figure 1.1 - this situation represents an instantaneous pertubation to the level in the well - there has been no time to develop a drawdown curve. This situation reflects a 'slug' test (aka a falling or rising head test) - ie where water (or air) is used to suddenly displace water in the well and, through monitoring its recovery, an estimation of the hydraulic conductivity ''k' can be made, as described below.

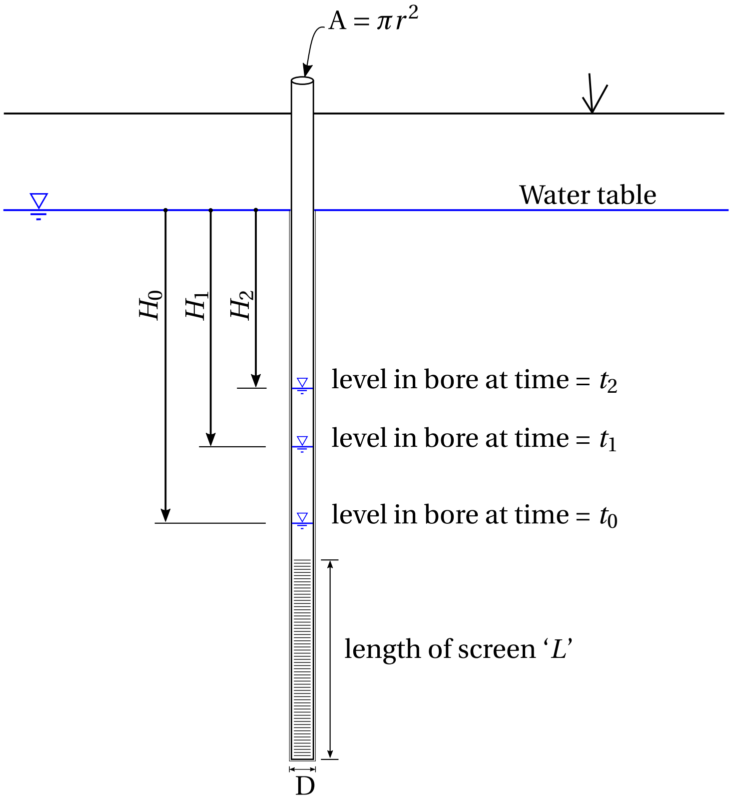

Figure 1.1 Flow into a well after sudden change in standiing water level

1.2 Analysis

Considering Figure 1.1, at time 't' the discharge 'Q_t' can be expressed as:

$$Q_t \, \mathrm{dt} = -A \, \mathrm{dH} \tag{1.2.1}$$

With substitution of Equation 1.1.1:

$$FkH_t \, \mathrm{dt} = -A \, \mathrm{dH} \tag{1.2.2}$$

Consider the change in water level between two times t_1 and t_2. Equation 1.2.2 can be arranged and integrated as:

$$\int_{t_1}^{t_2} \frac{Fk}{A} \mathrm{dt} = -\int_{H_1}^{H_2} \frac{1}{H} \mathrm{dH} \tag{1.2.3}$$

Giving:

$$\frac{Fk(t_2 - t_1)}{A} = -\ln(\mathrm{H}_2) + \ln(\mathrm{H}_1)$$
$$\frac{Fk(t_2 - t_1)}{A} = \ln\left(\frac{\mathrm{H}_1}{\mathrm{H}_2}\right)$$
rearranging :
$$k = \frac{A}{F(t_2 - t_1)} \ln\left(\frac{\mathrm{H}_1}{\mathrm{H}_2}\right) \tag{1.2.4}$$

Given the measurement of time elapsed between two well levels H_1 and H_2, an estimation of 'k' can be made using Equation 1.2.4.

In practice, the recovery of water levels in the well are continuously monitored with a logger, and can be plotted as per Figure 1.2, where the y-axis is $\frac{H_t}{H_0}$ and the x-axis is time. When plotted in this way, and with a y-axis as a logarithm, the observations should form a straight line. A line through the points, as shown, can be used to select a t_1 and t_2 and corresponding H_1 and H_2 which appropriately characterise the line and, substitution of these values into Equation 1.2.4 yields an estimation of k.

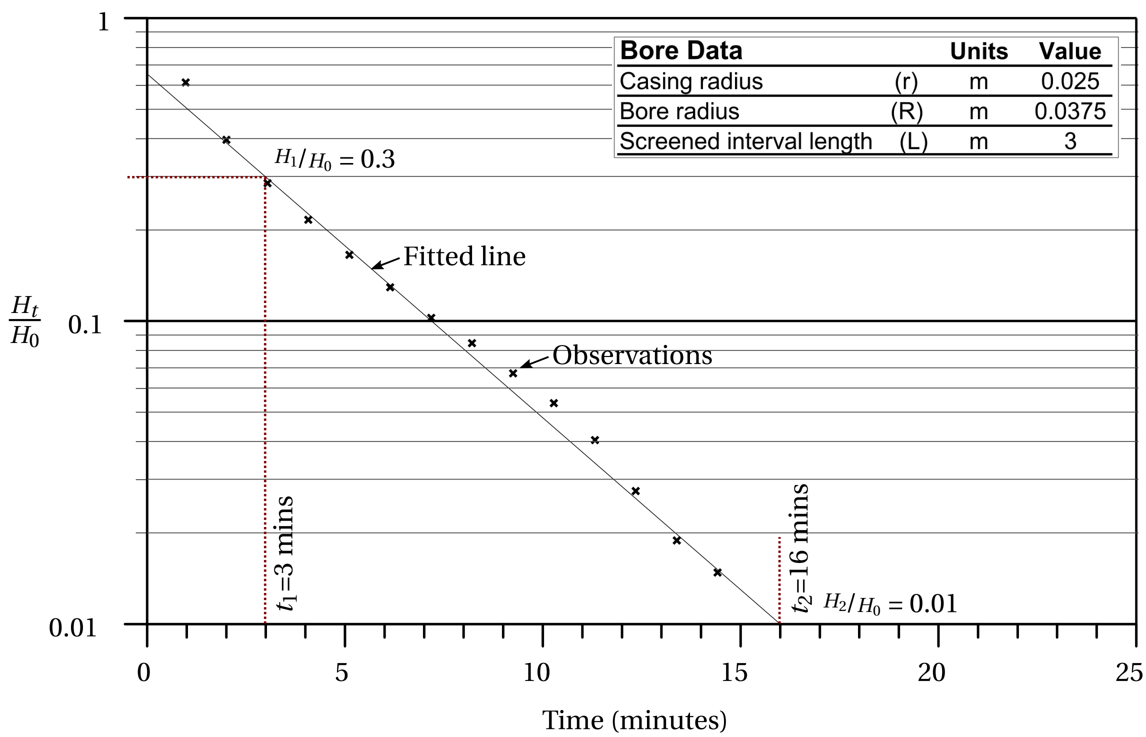

Figure 1.2 Example 1 of test data and interpretation

From Figure 1.2, substitution of the values for $t_1 = 3$ minutes (180 seconds) and $t_2 = 16$ minutes (960 seconds) and corresponding $H_1/H_0 = 0.3$ and $H_2/H_0 = 0.01$ into Equation 1.2.4 gives:

$$k = \frac{0.002}{4.3(960 - 180)} \ln\left(\frac{0.3\mathrm{H}_0}{0.01\mathrm{H}_0}\right)$$
$$= 2 \times 10^{-6}\ \mathrm{m.s}^{-1}$$

1.2.1 The Hvorslev 1951 'time-lag' solution

Hvorslev (1951) would not have had access to modern calculators, and perhaps sought to simplify the solution. Noting that $\ln\left(\frac{1}{0.37}\right) \approx 1$, Hvorslev (1951) defined a time-lag 'T', being the time taken for water level to move from its position H_0 at time $t = 0$ to a position H_T, where $H_T = 0.37H_0$ (i.e head has recovered 37%). Substitution of these conditions into Equation 1.2.4 gives:

$$\begin{aligned} k &= \frac{A}{F(t_{0.37} - t_0)} \ln\left(\frac{\mathrm{H_0}}{\mathrm{H_T}}\right) \\ &= \frac{A}{FT} \ln\left(\frac{\mathrm{H_0}}{0.37\mathrm{H_0}}\right) \\ &= \frac{A}{FT} \ln\left(\frac{1}{0.37}\right) \\ &= \frac{A}{FT} \end{aligned} \tag{1.2.5}$$

Noting $A = \pi r^2$ (r is the inside diameter of the well), and using the shape factor from Equation 1.1.3 gives:

$$k = \frac{r^2}{2LT} \ln\left(\frac{\mathrm{L}}{\mathrm{D}} + \sqrt{1 + \left(\frac{\mathrm{L}}{\mathrm{D}}\right)^2}\right) \tag{1.2.6}$$

$$\begin{aligned} &\approx \frac{r^2}{2LT} \ln\left(\frac{\mathrm{2L}}{\mathrm{D}}\right) \\ &\approx \frac{r^2}{2LT} \ln\left(\frac{\mathrm{L}}{\mathrm{R}}\right) \end{aligned} \tag{1.2.7}$$

The Hvorslev solution (Equation 1.2.7) is numerically simpler, but has the disadvantage that it forces the solution to fit to the early part of the curve - being the first 37% of recovery (ie between time $t = 0$ and time t when $H = 0.37H_0$). In many tests (eg Figure 1.3) this doesn't raise a problem. However, in some test conditions, the early part of the curve can be effected by initial flow through the gravel pack and the 'time-lag' solution, being fitted to this part of the curve, does not give an estimate of k for the formation. An example of such a test result is shown in Figure 1.3.

Solutions to the test in Figure 1.3 using Hvorslev (Equation 1.2.7) and using Equation 1.2.4 presented below demonstrate how largely different estimates of k arise by fitting to different parts of the curve.

From Figure 1.3, the Hvorslev (1951) 'time-lag' = 0.8 mins (48 seconds). From Equation **??**:

$$\begin{aligned} k &= \frac{0.002}{4.3 \times 48} \\ &\approx 1 \times 10^{-5} \ \mathrm{m.s^{-1}} \end{aligned} \tag{1.2.8}$$

From Figure 1.3, substitution of the values for $t_1 = 0$ minutes (0 seconds) and $t_2 = 10$ minutes (600 seconds) and corresponding $H_1/H_0 = 0.17$ and $H_2/H_0 = 0.0081$ into Equation 1.2.4 gives:

$$\begin{aligned} k &= \frac{0.002}{4.3(600 - 0)} \ln\left(\frac{0.16\mathrm{H_0}}{0.0083\mathrm{H_0}}\right) \\ &\approx 2 \times 10^{-6} \ \mathrm{m.s^{-1}} \end{aligned}$$

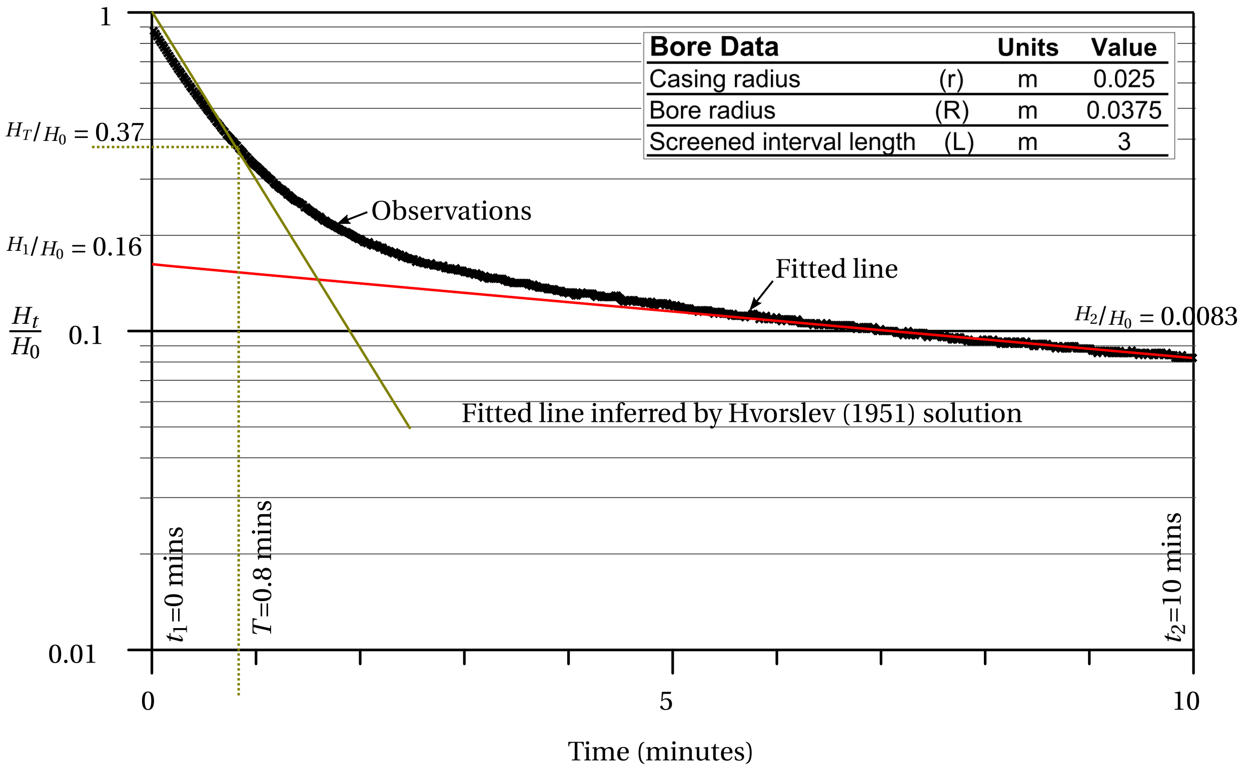

Figure 1.3 Example 2 of test data and interpretation