

## Guidance on the calculation of stream power dissipation for rock scour assessments

Steven E. Pells<sup>1</sup>, William L. Peirson<sup>2</sup>, Fanar Al-Qassab<sup>1</sup>

<sup>1</sup>PSM, G3 56 Delhi Road, North Ryde, NSW, 2113, Australia; e-mail: [steven.pells@psm.com.au](mailto:steven.pells@psm.com.au)

<sup>2</sup>Master, New College, UNSW Sydney, Kensington, NSW, Australia

### ABSTRACT

Various published methods for assessing scour of rock use stream power dissipation as an index of hydraulic erosive power. This paper presents recommended procedures for calculation of stream power dissipation to ensure compatibility with these published methods. Discussion on the usage of CFD modelling results in these comparative erosion assessments is also provided.

### INTRODUCTION

Unit stream power dissipation ( $\Pi_{UD}$ ) is applied as an index of erosive power in various published methods of rock scour analysis (eg van Schalkwyk et al 1994; Annandale, 1995; Kirsten et al 2000, Pells et al 2016, Douglas et al 2018). In these methods,  $\Pi_{UD}$  is used as an index to compare the hydraulic conditions at a site in question against case studies. There exist diverging practices in industry in the analysis and calculation of  $\Pi_{UD}$  for gradually- or rapidly-varied flow or in the presence of plunging flow conditions. Inappropriate or incorrect estimation of  $\Pi_{UD}$  will lead to erroneous estimations of rock scour when applying these methods. In addition, computational fluid dynamics (CFD) models allow various options for calculation of  $\Pi_{UD}$  that may yield estimates that are inconsistent with the estimations underpinning existing published rock scour assessment methods. This paper sets out the fundamental principles of  $\Pi_{UD}$  and presents calculation techniques that yield values consistent with the original intent of existing ‘comparative’ scour assessment methods.

### STREAM POWER DISSIPATION

A body of water contains hydro-mechanical energy by virtue of its mass and its velocity. In flowing water, hydro-mechanical energy may be dissipated through friction and turbulence production, and the average rate of this energy dissipation is observed as the gradient of the total energy line, referred to as the ‘energy slope’  $S_f$ . For flow in open, wide, channels this energy dissipation may be expressed as a power by multiplication with the mass discharge (and gravity):

$$\Pi_{UD} = \rho g \frac{Q}{B} \frac{\Delta H}{\Delta L} = \rho g q S_f \quad (1)$$

Where:  $\Pi_{UD}$  is the unit stream power dissipation ( $\text{W}\cdot\text{m}^{-2}$ )

$\rho$  is the water density ( $\text{kg}/\text{m}^3$ )

$g$  is the acceleration due to gravity ( $\text{m}\cdot\text{s}^{-2}$ )

$Q$  is the discharge ( $\text{m}^3\cdot\text{s}^{-1}$ )

$B$  is the channel width (m)

$q$  is the specific discharge ( $\text{m}^2\cdot\text{s}^{-1}$ )

$\frac{\Delta H}{\Delta L}$  = the loss in total hydraulic energy head  $\Delta H$  over a stream-wise distance  $\Delta L$   
 =  $S_f$ , the gradient of the total energy line.

$\Pi_{UD}$  is referred to as a power *dissipation* as it describes the rate of loss of hydro-mechanical power in the stream-wise direction. By using the specific discharge,  $\Pi_{UD}$  provides a measure of the power dissipated in the total water column per unit area of channel bed.

## USAGE OF STREAM POWER DISSIPATION IN ASSESSMENT OF EROSION

Dissipation of hydro-mechanical energy through friction and turbulence relates to the interaction of flow with the channel geometry. The dissipation of hydro-mechanical power has therefore been postulated to be indicative of the erosive power of water. Various geomorphologic studies have used stream power dissipation as an index of erosion processes or sediment transport capacity (e.g. Leopold *et al* 1964; Bagnold, 1966). Unit stream power dissipation ( $\Pi_{UD}$ ) has also been used for estimation of rock mass erosion. Annandale (1995) compiled a dataset of case studies of erosion in soil and rock materials and estimated an erosion threshold using  $\Pi_{UD}$  as a proxy for erosive power and Rock Mass Index called the Kirsten Index (Kirsten, 1982) to estimate erodibility of the earth or rock materials. To obtain estimates of  $\Pi_{UD}$  Annandale (1995) assumed uniform flow conditions (i.e.  $S_f = S_0$ , the bed slope) for channel flow cases and used classical analytical closed-form solutions for hydraulic jumps and for flows over knickpoints and drop structures. Case studies of erosion of unlined spillways were also assessed in terms of the Kirsten Index and  $\Pi_{UD}$  by van Schalkwyk *et al* (1994) and Kirsten *et al* (2000). Van Schalkwyk *et al* (1994) assumed uniform flow conditions but for plunging flow conditions assumed  $S_f = 3$ . Pells (2016) compiled a dataset of rock mass erosion cases through inspections of up to 30 spillways. Pells (2016) noted that uniform flow conditions seldom developed in the spillways examined and estimated  $\Pi_{UD}$  in each case considering gradually-varied flow (GVF) and rapidly-varied flow (RVF) conditions as resolved from 1D hydrodynamic modelling and classical closed-form solutions. This data confirmed a useful correlation between  $\Pi_{UD}$ , the observed erosion and various Rock Mass Indices.

These studies each presented methodologies which have proven to be useful to industry in assessment of spillway erosion. To use these methods, the practitioner makes an estimate of  $\Pi_{UD}$  and the relevant Rock Mass Index for the site in question and obtains an estimate of erosion vulnerability by comparison against corresponding values in the assembled case studies. These studies are referred to collectively in this present paper as “Comparative Methods”. All of these Comparative Methods noted that  $\Pi_{UD}$  is not a direct measure of erosive power but may be

considered as a proxy for the various processes and turbulent fluctuations that are thought to cause erosion. It is used as an index to allow comparison. Hence it is necessary that, when using these Comparative Methods, a compatible method for estimation of  $\Pi_{UD}$  is used.

This present paper considers the assessment of  $\Pi_{UD}$  for usage in these Comparative Methods. The estimation of other hydraulic parameters as a direct measure of erosive power or coupled kinematic analyses of erosion are beyond the scope of this present paper.

## **GUIDANCE ON APPROPRIATE CALCULATION OF STREAM POWER DISSIPATION**

The analytical and numerical analyses used in Comparative Methods adopted 1D flow assumptions – i.e. depth-averaged flow. The following principles can be followed to obtain estimates of  $\Pi_{UD}$  for Comparative Methods using such depth-averaged approaches. A separate discussion is then made on the usage of non depth-averaged Computational Fluid Dynamic (CFD) model results for estimation of compatible  $\Pi_{UD}$  values.

### **Peak, steady flows**

In their assembly of case data, the published Comparative Methods all assessed  $\Pi_{UD}$  at the peak discharge of the historical flood hydrograph and related this to the extent of erosion observed. Steady flow conditions were assumed. Clearly hydrographs differ in duration and pattern, and the extent of erosion cannot be attributed entirely to the instant of peak discharge. Pells (2016) assessed the total energy dissipation over the duration of various spill hydrographs and found that  $\Pi_{UD}$  at the peak of the hydrograph was broadly representative of the character of many hydrographs, particularly for ungated (uncontrolled) spills. This highlights how  $\Pi_{UD}$  is used only as a basis for comparison rather than as a physics-based process model. Practitioners should exercise judgement in selecting an appropriate discharge for representation of the site in question.

### **Use GVF analysis and plot the total energy line**

When preparing estimates of  $\Pi_{UD}$  it is recommended to undertake GVF analyses to plot the total energy line over the entirety of the spillway for the assessed flow condition, noting that the total energy line must consistently reduce in a stream-wise direction. This process demonstrates that an energy balance is maintained over the spillway domain, ensuring against selection of unrealistically large  $S_f$  values at any location. For example, unrealistically large values of  $S_f$  at one location would leave insufficient energy available to explain flow conditions further downstream. This analysis also indicates that in many spillways there is insufficient consistency of geometry for uniform flow conditions to develop. A uniform-flow assumption will over-estimate  $\Pi_{UD}$  in locations where uniform flow conditions have not yet developed, and may under-estimate  $\Pi_{UD}$  in locations of sudden change to roughness or geometry. A GVF analysis

over the entire spillway such as presented in Figure 1 can be readily achieved using a 1D hydrodynamic model such as HEC-RAS.

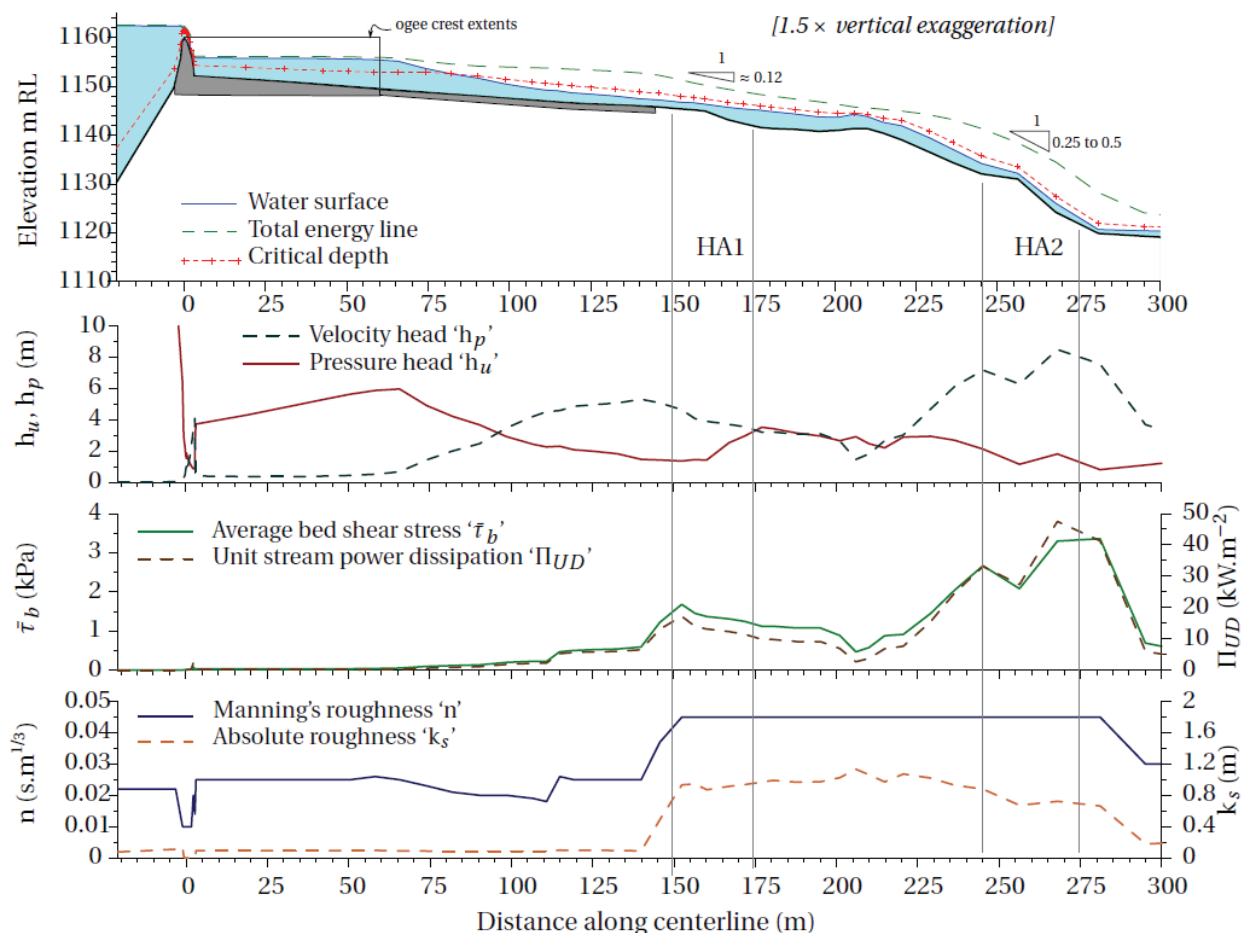


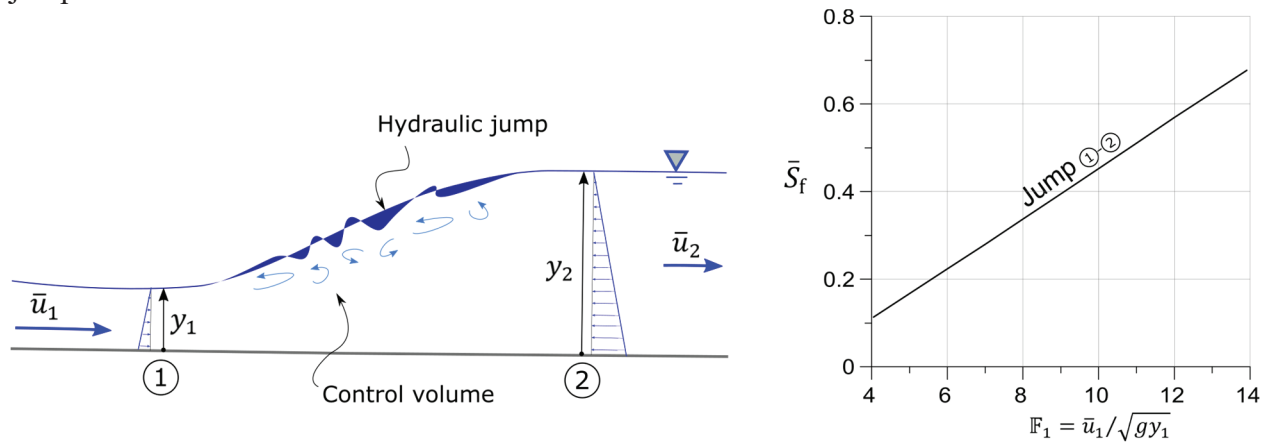
Figure 1. Example of estimation of  $S_f$  and  $\Pi_{UD}$  from steady GVF analyses over the spillway (from Pells, 2016).

### Revise estimates for RVF where suitable

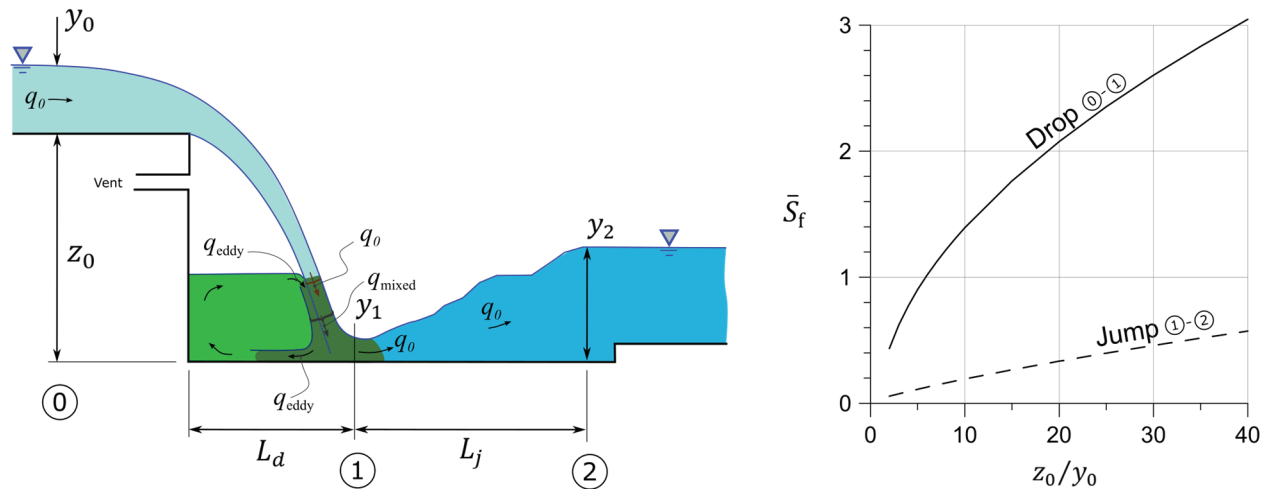
GVF analyses will not accurately resolve very steep sections or locations of RVF, such as hydraulic jumps or plunging flow conditions. In such locations, the estimations of  $S_f$  can be refined using closed-form analytical (ie momentum-based) solutions or observations from physical or CFD modelling. A classical momentum solution to hydraulic jumps (i.e. such as presented in Chow, 1959 and Henderson, 1966) allows estimation of the loss of energy from the start to the end of the jump (i.e.  $\Delta H$ ). The estimation of  $S_f$  over the jump (as a function of the Froude number of approaching flow) presented in Figure 2 is obtained by taking the length of the jump (ie  $\Delta L$ ) as 6 times the downstream water depth (Peterka, 1957; Henderson, 1966). A similar plot is prepared in Figure 3 for the case of flows plunging over a drop structure

(Henderson, 1966). From these analyses it is seen  $S_f < 0.7$  for most hydraulic jump conditions and  $S_f < 3$  for most plunging flow conditions relevant to dams.

These solutions can guide the estimations of  $S_f$  and hence  $\Pi_{UD}$  in some cases of RVF, although they are suitable only to these specific geometries and tailwater conditions. For example, the solution in Figure 3 is not applicable to all cases in Figure 4. It is also noted that  $S_f$  may not be linear between locations where total energy is calculated. For example, this uncertainty led Pells (2016) to adopt a cautious assumption that 80 % of the energy is lost in first 50% of a hydraulic jump.



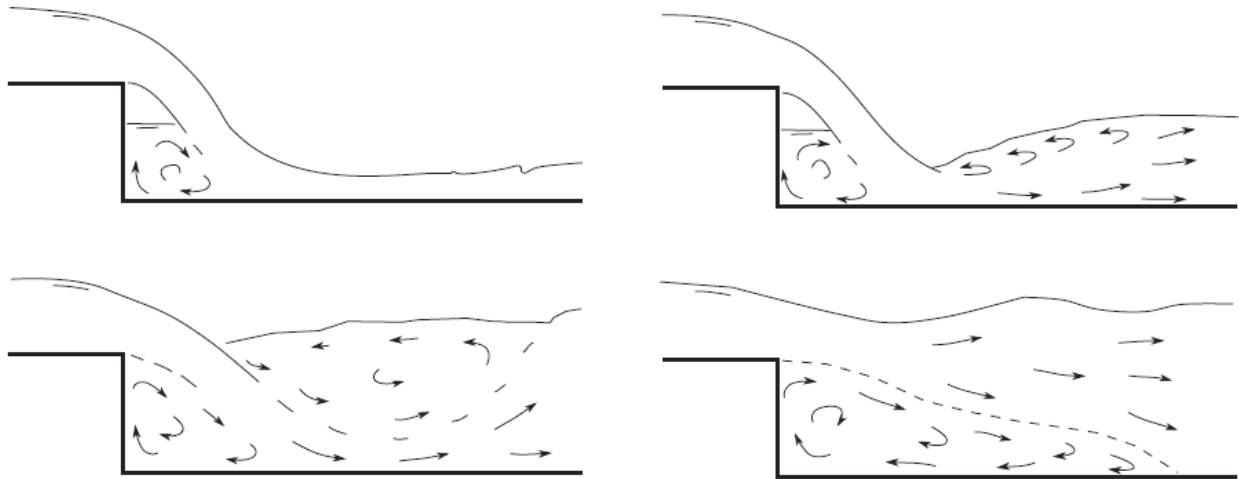
**Figure 2. Classical Analytical solution to  $S_f$  over a hydraulic jump eg Henderson, 1966 (worked calculations presented in Pells, 2016)**



**Figure 3. Classical analytical solution to  $S_f$  over a drop structure eg Henderson, 1966 (worked calculations presented in Pells, 2016)**

The present writers have encountered various times in which engineering consultants and publications have assessed stream power dissipation for plunging flow conditions by assuming that the entire energy head in the plunging jet is dissipated upon impact – ie  $\Delta H$  is taken as the

drop height and  $\Delta L$  is taken as the thickness of the jet. These assumptions do not consider: the losses in energy during the plunge trajectory prior to impact; the energy remaining on the flow after impact, or; the area over which energy is reasonably dissipated. Such assumptions infer  $S_f$  values well in excess of those resolved from momentum analyses and result in estimates of  $\Pi_{UD}$  which are unrealistically high. In cases of plunging flow such as illustrated in Figure 3 (i.e. resulting in non-submerged flow conditions downstream) it is recommended to estimate  $S_f$  from Figure 3, or otherwise through specific physical model or CFD studies.



**Figure 4. Flow conditions over a drop for various tailwater conditions (adapted from Vischer and Hager, 1995).**

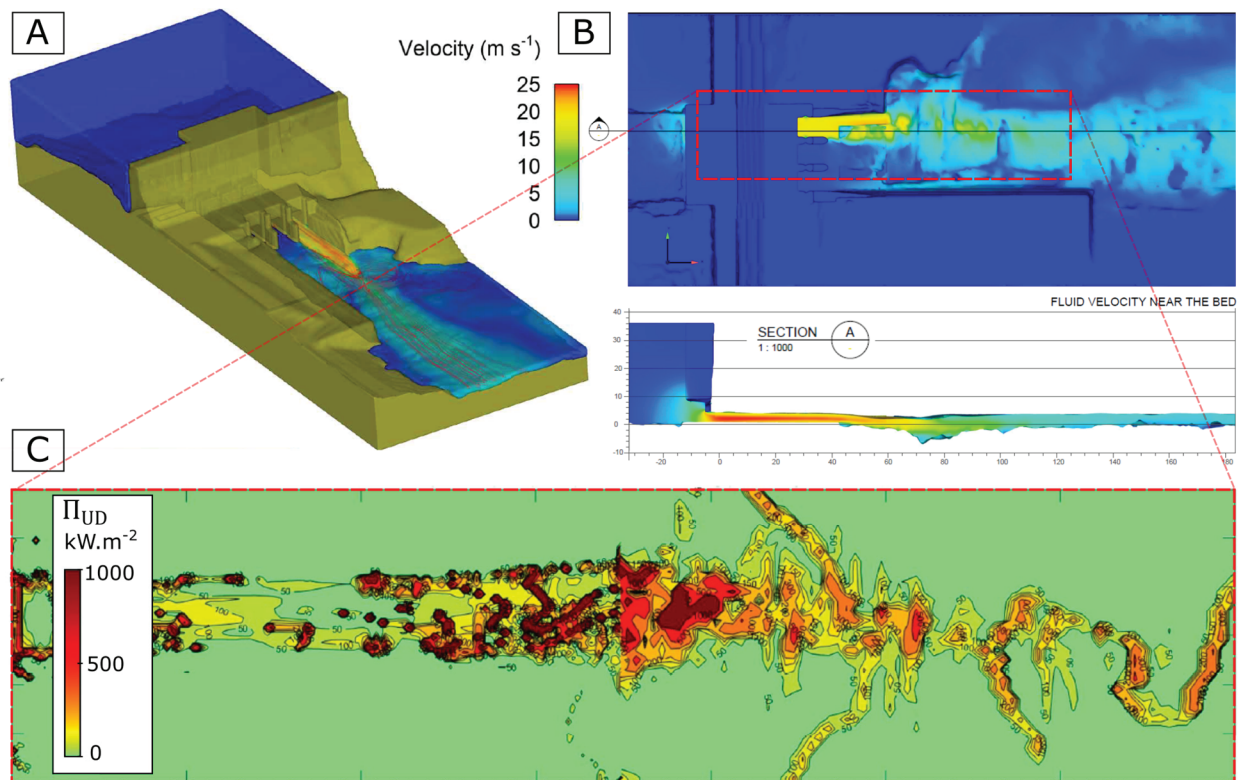
### **Be cautious of extreme values**

Dams with larger design flows tend to feature wider spillways, such that the specific discharge for historical floods, or even design floods, is typically less than  $50 \text{ m}^2/\text{s}$ . In fact, the present authors are of the view that specific discharges greater than  $50 \text{ m}^2/\text{s}$  can itself be used as a warning of potentially higher scour vulnerability. With consideration of the typical values of  $S_f$  resolved in Figures 1 and 2, or typically spillway slopes (using a uniform flow assumption as a rough guide), it can be seen that values of  $\Pi_{UD}$  will typically be expected to be  $< 1000 \text{ kW/m}^2$ . Larger values may be an indication of unrealistic  $S_f$  values, non-spatially averaged estimates or errors in calculation.

### **Comments on calculating stream power dissipation from CFD results**

The steady, depth-averaged analyses presented above yield an estimation of  $\Pi_{UD}$  which is temporally- and spatially- averaged over the width of the channel and the flow depth. The definition of  $\Pi_{UD}$  in Equation 1 is the average power dissipation across the depth of the water column – i.e., in plan view. Commercial CFD packages from Flow Science (Flow3D), Ansys (Fluent and CFX) and Siemens (Star CCM+) do not include in-built functions for directly reporting  $\Pi_{UD}$ . Analysis of power dissipation can be calculated by post-processing CFD results, such as: integrating turbulent dissipation in each computational cell; the product of shear stress and velocity (accounting in some manner for a “hydraulic radius” of a cell), or; resolving

Equation (1) for each computational cell. Such analyses can yield detailed maps of stream power dissipation over the spillway domain and in 3 dimensions. An example of such analysis from CFD results is presented in Figure 5. This presentation shows high spatial and temporal variability. This presentation is useful in highlighting regions of higher hydraulic loading. However, Comparative Methods strictly only provide comparative guidance on erosion based on spatially and temporally averaged values of  $\Pi_{UD}$  from depth-averaged analyses. The “hot spots” of high dissipation in the CFD results suggest localized and severe erosion in places where a corresponding analysis using depth-averaged assumptions showed no such risk.



**Figure 5. Example of  $\Pi_{UD}$  calculated from CFD results. A) 3D perspective of the spillway, showing velocities. B) Plan and sectional views of velocities C) Plan view of unit stream power dissipation where Equation 1 is applied to each computational cell, and the values summed through the flow depth.**

Spatial and temporal averaging of CFD results is required to estimate  $\Pi_{UD}$  values that can be applied in Comparative Methods. In the software Flow3D™ this may be achieved by reporting the total head and discharge at various “flux planes” and using these results to assess  $S_f$  as the difference in total head between two sequential flux sections (ie  $\Delta H$ ) divided by the stream-wise distance between the sections (ie  $\Delta L$ ), thereafter applying Equation (1) to estimate  $\Pi_{UD}$ . This methodology reduces detailed 3D model results to a simpler 1D-style representation of flow characteristics and does not utilize the greater potential of a CFD model, but it is required to ensure estimates of  $\Pi_{UD}$  are compatible with published Comparative Methods.

## CONCLUSION

Unit stream power dissipation ( $\Pi_{UD}$ ) can be used as an index of erosive power. It is used in various publications for assessing erosion vulnerability of unlined spillways through comparison against case studies of historical erosion. Guidance on calculation of  $\Pi_{UD}$  is presented in this present paper to maintain validity with the case study data in currently published methods. Practitioners are encouraged to assess dissipation of energy over the entire spillway reach as considering hydraulic features in isolation can lead to spurious dissipation estimates. Classical solutions to RVF conditions indicate that for most dam spillways  $S_f$  will be significantly less than 3 and extreme prototype  $\Pi_{UD}$  values seldom exceeded  $1000 \text{ kW}\cdot\text{m}^{-2}$ . CFD models can provide detailed perspectives of stream power dissipation, but spatial and temporal averaging is required to obtain estimates suitable for comparison to published case study data.

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